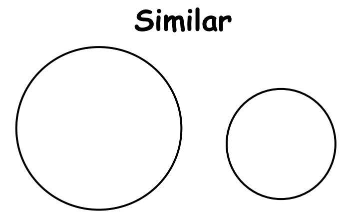
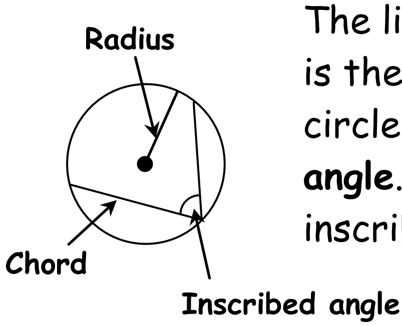
Name: ____________________ Date:_______________

Circles: Theorems about Circles

To be **similar**, two objects do not need to have the same size, but must have the same shape. In order for something to be a **circle**, it must have a center that is equidistant to any point on its circumference. Therefore, all circles are similar.

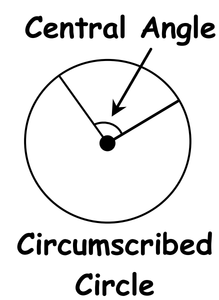


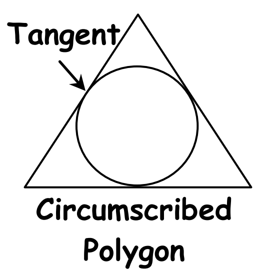


The line connecting the center to the circumference of the circle is the **radius**. A **chord** is a segment with endpoints that lie on the circle. Combining two chords within a circle creates an **inscribed angle**. The vertex of an inscribed angle rests on the circle. An inscribed angle that rests on the diameter is a right angle.

Combining chords into a polygon creates a **circumscribed circle**.

Combining two radii creates a **central angle**. The vertex of the central angle rests on the center of the circle.

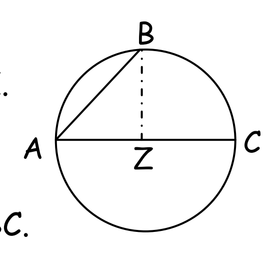




A **tangent** is a line that is in the same plane as a circle and intersects the circle at exactly one point. The tangent of a circle is always perpendicular to the radius. In a **circumscribed polygon,** the sides of the polygon are made up of the tangents of a circle.

Challenge: Given that $\angle ABC$ is inscribed in circle Z, prove that $m\angle ABC$ is half the measure of $\widehat{AC}$.

Step 1: Draw BZ. $m\widehat{AC} = m\angle AZC$;
Step 2: Use Exterior Angle Theorem $m\angle AXC = m\angle ABZ + m\angle BAZ$.
Step 3: Since $\overline{ZA}$ and $\overline{ZB}$ are radii, $\overline{ZA} \cong \overline{ZB}$, then ΔAZB is isosceles.
Thus, $m\angle ABZ = m\angle BAZ$
Step 4: Substitution $m\widehat{AC} = 2m\angle ABZ$ or $2m\angle ABC$.
Thus, $\frac{1}{2} m\widehat{AC} = m\angle ABC$.



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Sketch.

1. Inscribed equil. Δ, b=5

2. Circumscribed Δ, r=3

3. Inscribed quadrilateral

4. Tangent AB; Intersects cir. X at point B.

Inscribed Quadrilateral Theorem: If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary

Given: LMNO is inscribed in circle T; m∠N = 105; m∠O =96
Prove: m∠L = 75 and m∠M = 84

1. ______________________________ 1. Given

2. m∠L + m∠N = 180 and m∠M + m∠O = 180 2. ____________________________

3. ______________________________ 3. Substitution

4. _____________________________ 4. Addition property of equality

5. m∠L = 75 and m∠M = 84 5. ____________________________

Bonus: Explain in your own words why the opposite angles of an inscribed quadrilateral are supplementary.

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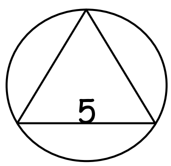
Answer Key

Circles: Theorems about Circles

Sketch.

1.



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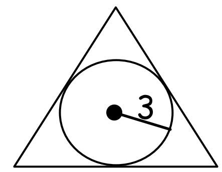

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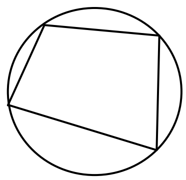

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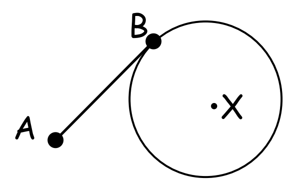

Prove

1. LMNO is inscribed in circle T. M∠N = 105; m∠O = 96
2. Inscribed Quadrilateral Theorem
3. m∠L + 105 = 180; m∠M + 96 = 180
4. 180-105 = m∠L; 180-96 = m∠M
5. Simplify

Bonus:

The sum of all angles of a quadrilateral is equal to 360 degrees. The sum of any two opposing angles would therefore be supplementary.