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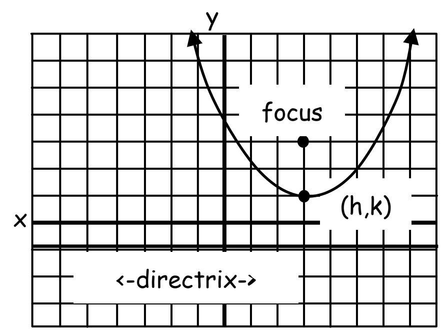
Expressing Properties: Conic Sections

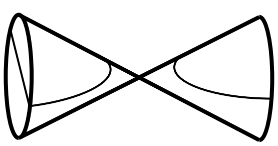
There are many ways to slice a cone. As we look at conic sections, we discover a few interesting geometric properties that are produced by the intersection of a plane and a cone or cones.

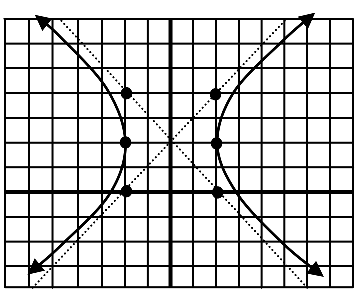
A parabola



A focus point of a parabola is equidistant from a focus point and the directrix (which is a fixed line). The vertex is always halfway between the focus and directrix at a distance of p from both. There are two equations used, depending on the orientation of the axis of symmetry:

<u>Horizontal Axis</u>	<u>Vertical Axis</u>
Focus: (h+p, k)	Focus: (h, k+p)
Equation:	Equation:
$(y-k)^2 = 4p(x-h)$	$(x-h)^2 = 4p(y-k)$





A hyperbola

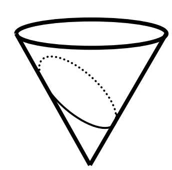
A hyperbola is formed by a plane cutting through two cones. Hyperbolas can have foci on the x-axis (the curves open up and down) or the y-axis (the curves open to the right and left).

The equations may be expressed one of two ways:

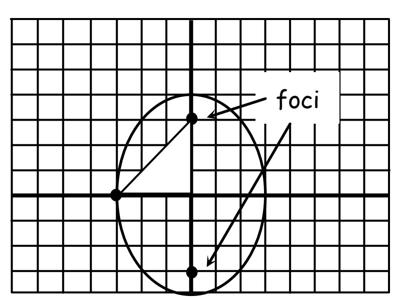
<u>Foci on the y-axis</u>	<u>Foci on the x-axis</u>
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The asymptotes (dash line) pass through points (+a,-b) and (-a,+b) as well as (a,b) and (-a,-b) starting at the center point.

Ellipses



An ellipses is formed by the intersection of a cone and a plane at an angle of > or < 180 parallel to the base of the cone. The shape rendered is a “squashed” circle, or an oval.



Ellipses have a focus, a center, and vertices. The center is equidistant vertically and equidistant horizontally. The foci can be found using the equation $a^2 + b^2 = c^2$. Look familiar? It’s Pythagorean Theorem!

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We can use Pythagorean Theorem to find a focus if we know the center (*h, k*), a vertex, and the distance between a vertex and a focus (*c*).

Standard Equation

$$\frac{(x-h)^2}{r_x^2} - \frac{(y-k)^2}{r_y^2} = 1$$

The standard equation: r_x is the distance from the center of the circle in the x direction and r_y is the distance from the center in the y direction.

The focus of an ellipse is distance *c*, and its equation depends on the orientation of its major axis, or its longest length.

Horizontal	Vertical
$r_x^2 - r_y^2 = c^2$	$r_y^2 - r_x^2 = c^2$

Practice. Identify the conic section described.

1. A plane that intersects vertically with two stacked cones.

2. A plane that intersects diagonally with a single cone, so that it does not form a closed curve.

3. A plane that intersects diagonally with a single cone, so that it forms a closed curve.

4. A plane that intersects parallel to the base of a single cone, so that it forms a closed curve.

5. A plane that intersects a cone at a 45 degree angle, forming a closed curve.

6. The intersection of a single cone and a plane at a degree of 30. The image produced is an open curve.

7-10. Identify the equation of the following conic sections.

7. Circle with center (0,3), radius 5

8. Hyperbola: a=2, b=3, h=0, k=-3
   Foci is on the x-axis

9. Horizontal Ellipse: c=5, r_x=4; center: (0,4)

10. Vertical Ellipse: center: (5,2); r_x=3, c=5

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Answer Key

Geometric Properties: Conic Section

1. Hyperbola
2. Parabola
3. Ellipse
4. Circle
5. Ellipse
6. Parabola
7. $x^2 + (y-3)^2 = 25$
8. $\frac{x^2}{4} - \frac{y^2}{9} = 1$
9. $\frac{x^2}{16} - \frac{(y-4)^2}{9} = 1$
10. $\frac{(x-5)^2}{16} - \frac{(y-2)^2}{9} = 1$