Circles: Theorems about Circles

To be similar, two objects do not need to have the same size, but must have the same shape. In order for something to be a circle, it must have a center that is equidistant to any point on its circumference. Therefore, all circles are similar.

The line connecting the center to the circumference of the circle is the radius. A chord is a segment with endpoints that lie on the circle. Combining two chords within a circle creates an inscribed angle. The vertex of an inscribed angle rests on the circle. An inscribed angle that rests on the diameter is a right angle.

Combining chords into a polygon creates a circumscribed circle.

Combining two radii creates a central angle. The vertex of the central angle rests on the center of the circle.

A tangent is a line that is in the same plane as a circle and intersects the circle at exactly one point. The tangent of a circle is always perpendicular to the radius. In a circumscribed polygon, the sides of the polygon are made up of the tangents of a circle.

Challenge: Given that \( \angle ABC \) is inscribed in circle \( Z \), prove that \( m \angle ABC \) is half the measure of \( \widehat{AC} \).

Step 1: Draw \( BZ \).
Step 2: Use Exterior Angle Theorem
Step 3: Since \( ZA \) and \( ZB \) are radii, \( ZA \cong ZB \), then \( \triangle AZB \) is isosceles.
Step 4: Substitution

Thus, \( m \angle ABZ = m \angle BAZ \)
\[ m \angle AC = 2m \angle ABZ \text{ or } 2m \angle ABC. \]
Thus, \( \frac{1}{2} m \angle AC = m \angle ABC. \)
**Sketch.**

1. Inscribed equil. $\Delta$, $b=5$

2. Circumscribed $\Delta$, $r=3$

3. Inscribed quadrilateral

4. Tangent $AB$; Intersects cir. $X$ at point $B$.

**Inscribed Quadrilateral Theorem:** If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

*Given:* $LMNO$ is inscribed in circle $T$; $m\angle N = 105$; $m\angle O = 96$

*Prove:* $m\angle L = 75$ and $m\angle M = 84$

1. ________________ 1. Given

2. $m\angle L + m\angle N = 180$ and $m\angle M + m\angle O = 180$

3. ________________ 3. Substitution

4. ________________ 4. Addition property of equality

5. $m\angle L = 75$ and $m\angle M = 84$

**Bonus:** Explain in your own words why the opposite angles of an inscribed quadrilateral are supplementary.
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Sketch.
1.  
2.  
3.  
4.  

Prove
1. LMNO is inscribed in circle T. ∠N = 105; ∠O = 96
2. Inscribed Quadrilateral Theorem
3. m∠L + 105 = 180; m∠M + 96 = 180
4. 180 - 105 = m∠L; 180 - 96 = m∠M
5. Simplify

Bonus:
The sum of all angles of a quadrilateral is equal to 360 degrees. The sum of any two opposing angles would therefore be supplementary.