Volume refers to the amount of space taken up by a three-dimensional object. Here is a brief listing of volume formulas.

<table>
<thead>
<tr>
<th>Item</th>
<th>Surface Area</th>
<th>Volume Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder</td>
<td>$B = \pi r^2$</td>
<td>$V = \pi r^2h$</td>
</tr>
<tr>
<td></td>
<td>$L = 2\pi rh$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$S = L + 2B$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sphere: $\pi r^3$</td>
</tr>
<tr>
<td>Regular Pyramid</td>
<td>$B = \ell \cdot w$</td>
<td>$V = Bh$</td>
</tr>
<tr>
<td></td>
<td>$L = \frac{1}{2}P\ell$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$S = \frac{1}{2} P\ell + B$</td>
<td></td>
</tr>
<tr>
<td>Cone</td>
<td>$\ell =$ slant height</td>
<td>$V = \pi rh$</td>
</tr>
<tr>
<td></td>
<td>$L = \pi r\ell$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$S = L + B$, or $\pi r\ell + \pi r^2$</td>
<td></td>
</tr>
<tr>
<td>Prism</td>
<td>$S = 2(\ell_1 \cdot w_1) + 2(\ell_2 \cdot w_2)$</td>
<td>$V = \ell \cdot w \cdot h$</td>
</tr>
<tr>
<td></td>
<td>$+ 2(\ell_3 \cdot w_3)$</td>
<td></td>
</tr>
</tbody>
</table>

A famous mathematician, Cavalieri, argued that if the cross-section of two three-dimensional objects consistently had the same area, then those objects have the same volume.

**Example:** Two cylinders are laid out side-by-side but look different. Explain how we can use Cavalieri’s principle to show they have the same volume.

**Answer:** A cross-section of both objects determines that they have the same area: $16\pi$. Cavalieri proposed that if a cross-section of two objects consistently had the same surface area, then the two objects would be the same volume.
Practice. Determine what information you will need in order to find the volume for each object.

1. [Diagram of a pyramid]

Solve. Find the volume of each figure.

3. Pyramid
   \[ h = 10 \]
   \[ B = 16 \]

4. Cone
   \[ h = 8 \]
   \[ r = 3.5 \]

5. Cylinder
   \[ h = 8 \]
   \[ C = 4\pi \]

6. Sphere
   \[ C = 8\pi \]

7. Rectangular Prism
   \[ h = 4 \]
   \[ l = 8 \]
   \[ w = 2 \]

8. Cube
   \[ w = 15 \]

Use Cavalieri's principle to determine whether the objects pictured have the same volume. Explain your answer.

9. [Diagram of two rectangular prisms]

10. [Diagram of two cylinders]
Answer Key

**Geometric Measurement: Using Volume Formulas**

1. Area of the base and height of the object
2. Height of the object and radius (or the area of the base)
3. 160
4. 87.92
5. 100.48
6. 200.96
7. 64
8. 3375
9. No. A cross-section of the objects would not consistently render a similar area throughout.
10. Yes. A cross-section of the objects would consistently render a similar area throughout.