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Expressing Properties: Coordinate Proofs

We can use coordinates to prove simple geometric theorems algebraically by using simple coordinate algebra. The formulas we will use are:

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example: Without graphing, determine whether points A(1,4), B(3,8), C(3,2), and D(1,-2) form rectangle ABCD.

Step 1: Find the slope of two sides of the figure. A slope is a measure of the rise/fall of a line on a coordinate plane. As a result, parallel lines have the same slope.

$m_{AB} = \dfrac{8 - 4}{3 - 1} = 2$

$m_{CD} = \dfrac{-2 - 2}{1 - 3} = 2$

$m_{AB} = m_{CD}$

$AB \parallel CD$

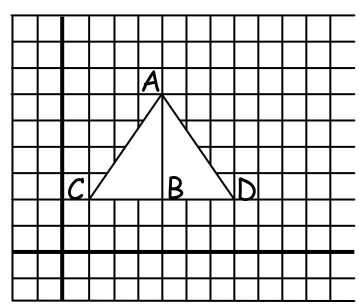
Step 2: Identify whether the sides of ABCD intersect at 90° angles. In order for an object to be a square, it must contain two sets of parallel lines that intersect at 90 degree angles. Coordinates that intersect at 90 degrees are considered perpendicular.

$m_{AB} = \dfrac{8 - 4}{3 - 1} = 2$

$m_{BC} = \dfrac{2 - 8}{3 - 3}$ = undefined (vertical line)

Perpendicular lines have slopes that are complete reciprocals both in sign (+/-) and placement. In order to be completely reciprocal to mAB, mBC would have to be - ½. mBC ≠ - ½; therefore ABCD is not a rectangle.

Example: Use the given coordinates to find the midpoint of segment AB. Then, determine the perimeter and area of ΔACD.



Step 1: Determine the length of AB/2. Midpt = 2

Step 2: Calculate the length of AC, CD, and DA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d_{AC} = \sqrt{(4 - 1)^2 + (6 - 2)^2}$$
$$d_{CD} = \sqrt{(1 - 7)^2 + (2 - 2)^2}$$
$$d_{AD} = \sqrt{(4 - 7)^2 + (6 - 2)^2}$$

AC = 5
CD = 6
AD = 5

Step 3: Calculate the Perimeter and Area of ΔACD.

P = s1 + s2 + s3 = AC + CD + AD = 5 + 6 + 5 = 16

$A = \frac{1}{2}Bh = (\frac{1}{2})(6)(4) = 12$

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Practice. Identify whether the following segments are parallel, perpendicular, or neither. Graph.

A (1, 4) B (2, 6) C (3, 3) D (2, 2) E (3, 0) F (3,4)

1. AB and DF

2. BD and AE

3. CD and AE

4. AF and BD

5. BD and CF

6. DF and AC

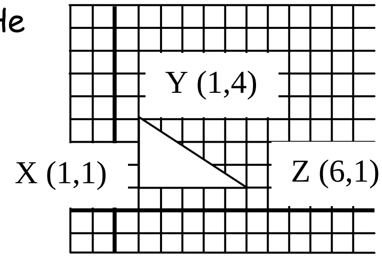
Use the distance formula to determine the perimeter and area of each of the following objects.

7. A (-2,1); B (2,4); C (6,1)
P ∆ABC = ______
A ∆ABC = ______

8. P(1,3); Q(2,5); R(4,4); S(3,2)
P □PQRS = ______
A□PQRS = ______

9. T(1,2); U(2,2); V(4,6); W(3,6)
P = ______
A = ______

10. A man is building a frame for the roof of his doghouse. He needs to add supports that will connect at the midpoints of XY, YZ and ZX. Calculate the coordinates for each connection.



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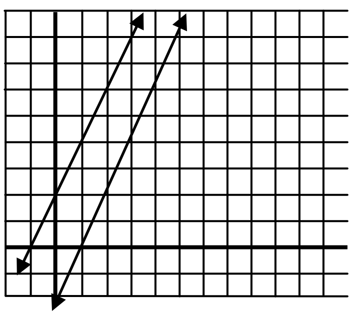
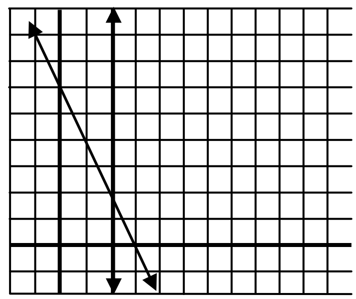
Answer Key

Expressing Properties: Coordinate Proofs

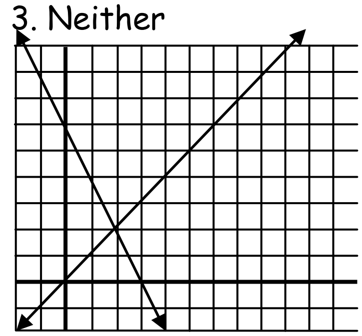
1. Parallel



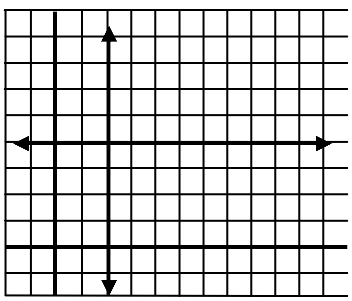
2. Neither



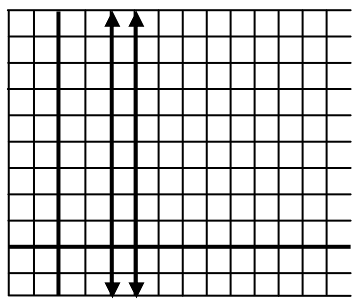
3. Neither



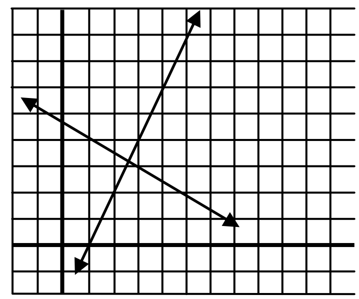
4. Perpendicular



5. Parallel



6. Perpendicular



7. A (-2,1); B (2,4); C (6,1)
P ΔABC = 18
A ΔABC = 16

8. P(1,3); Q(2,5); R(4,4); S(3,2)
P □PQRS = 4√8
A□PQRS = 128

9.T(1,2); U(2,2); V(4,6); W(3,6)
P = 10
A = 4

10. (1, 2.5); (3.5, 1); and (3.5, 2.5)