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Expressing Properties: Conic Section

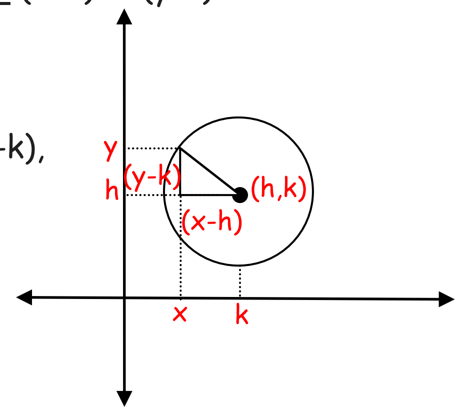
Conic Sections: The Circle

A *conic section* is a way of expressing the intersection of a plane with a cone. When a cone is intersected by a plane that is parallel to the base of the cone, the result is a circle.

Equations of circles

All points on a circle are equidistant from the center. We can use this information to write an equation with center (h, k) and radius r: <u>*Equation of a circle:*</u> $(x-h)^2 + (y-k)^2 = r^2$

We can use Pythagorean Theorem to find states the radius of a circle. Using the theorem $c^2= a^2 + b^2$, the radius can be represented by *c*. Sides *a* and *b* are represented by (x-h) and (y-k), respectively.



Example: The center of a circle is (-1,3) and passes through (-2, 6). Find the radius using the Pythagorean Theorem.

Step 1: Let (h, k) represent the center of the circle. $a^2 + b^2$ can be represented $(x+1)^2 + (y-3)^2 = r^2$.

Step 2: Substitute for values x and y

Step 3: Simplify and solve for *r*.

$r^2 = (-2+1)^2 + (6-3)^2$

$r^2 = (-1)^2 + (3)^2$

$r^2 = 1 + 9 = 10$

$r = \sqrt{10}$

Completing the square

When the center of a circle is unknown, we can complete the square to find its location.

Example: Find the center point of circle with equation $4x^2 - 8x + 16y + 4y^2 - 12 = 0$

Step 1: Verify that the squared terms have matching coefficients, $\underline{4}x^2$ and $\underline{4}y^2$

Step 2: Divide your coefficient from both sides of equal sign:

$$\frac{4x^2-8x+16y+4y^2-12}{4} = \frac{0}{4} \quad = \quad \frac{4x^2}{4} - \frac{8x}{4} + \frac{16y}{4} + \frac{4y^2}{4} - \frac{12}{4} = \frac{0}{4} \quad = \quad x^2 - 2x + 4y + y^2 - 3 = 0$$

Step 3: Subtract any remaining integers from both sides: $x^2 - 2x + 4y + y^2-3=0$

$x^2 - 2x + 4y + y^2 = 3$

Step 4: Divide each non-squared integer by 2 and square: $(-2/2)^2 =1$; $(4/2)^2= 4$

Step 5: Add your solutions to both sides of the equation: $x^2 - 2x + 1 + y^2 + 4y + 4 = 3$

Step 6: Factor: $(x - 1)^2 + (y + 2)^2 = 3$

From here, we can determine that the center of the circle is (1, -2) and it has a radius of $\sqrt{3}$

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Practice. Use the information provided to determine the equation of each circle.

1. Center *A* (4,2) and radius 2

2. Center *B* (0, -4) and radius 6

3. Center *B* (0,0) and radius 5

Practice. Use Pythagorean Theorem to determine the radius of each circle.

4. Center *A* (-5,-3) and point (1,-1)

5. Center *B* (-2, -8) and point (4, 2)

6. Center *A* (0, 0) and point (3, 6)

Practice. Complete the square to determine the equation of each circle.

7. $2y + y^2 - 6x + x^2 - 1 = 0$

8. $x^2 - 4x + 16y + 2y^2 - 2 = 0$

9. $2x^2 - 8y + 6x + 2y^2 - 2 = 0$

10. $y^2 + 4 - 16x + x^2 - 12y = 0$

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Answer Key

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1. $(x-4)^2 + (y-2)^2 = 4$
2. $x^2 + (y+4)^2 = 36$
3. $x^2 + y^2 = 25$
4. r=2
5. r=4
6. r=3
7. $(x-3)^2 + (y+1)^2 = 11$
8. Not a Circle
9. $(x+3)^2 + (y-2)^2 = 13$
10. $(x-8)^2 + (y-6)^2 = 104$